Calculus Q1 Quarter Test Review

1) $-\infty$ 2) ∞ 3) DNE (Left and Right are not the same)

4) 1 5) -1 6) ∞ 7) -2

8) $$f'(x) = \lim_{h\to 0} \frac{2(x+h)^2 - 8(x+h) + 5 - (2x^2-8x+5)}{h} = \frac{2(x^2+2xh+h^2) - 8x - 8h + 5 - 2x^2 + 8x - 5}{h}$$

$$\frac{2x^2+4xh+2h^2-8x-8h+5-2x^2+8x-5}{h} = \frac{4xh+2h^2-8h}{h} = 4x+2h-8$$

$h \to 0$ then $\boxed{4x-8}$

9) $f(x) = 5x + 2\sqrt[3]{x} - \frac{3}{x^2}$

$f(x) = 5x + 2x^{1/3} - 3x^{-2}$

$\boxed{f'(x) = 5 + \frac{2}{3}x^{-\frac{2}{3}} + 6x^{-3}}$

Power Rule Only

10) $f(x) = 5\sin(3x+1)$ Chain Rule

$f'(x) = \cos(3x+1)\cdot(3)$

$\boxed{f'(x) = 3\cos(3x+1)}$

11) $f(x) = \sqrt[4]{(x^2+5x)^3}$

$f(x) = (x^2+5x)^{3/4}$ Chain Rule

$f'(x) = \frac{3}{4}(x^2+5x)^{-\frac{1}{4}} \cdot (2x+5)$

12) $y = x\cdot\cos(x)$ Product rule

$y' = x(-\sin(x)) + (1)(\cos(x))$

$x = 0$

Point: $y = 0\cdot\cos(0) = 0$ $(0,0)$

Slope: $y' = 0(-\sin(0)) + 1(\cos(0))$

$= 0 + 1$

$= 1$

$\boxed{y - 0 = 1(x-0)}$ OR $\boxed{y = x}$

$@x = -1$

13) $y = 3x^2 - 2x + 1$ $\quad y' = 6x - 2$

Point: $(-1, 6)$ $\quad$ Slope: $y' = 6(-1) - 2 = -8$

$$\boxed{y - 6 = -8(x+1)}$$

14) $y = 5 - x^2$ $\quad y' = -2x$

Point: $y = 5 - (2)^2$ $\quad$ Slope: $y' = -2(2)$

$y = 1$ $\quad y' = -4$ ← slope of tangent

$(2, 1)$ $\quad \boxed{y - 1 = \frac{1}{4}(x-2)}$ $\quad \frac{1}{4}$ = slope of normal (negative reciprocal)

15) $h(x) = f(g(x))$

Chain Rule

$$h'(x) = f'(g(x)) \cdot g'(x)$$
$$h'(1) = f'(g(1)) \cdot g'(1)$$
$$\underbrace{f'(1)} \cdot (-1)$$
$$= (2)\ (-1)$$
$$= \boxed{-2}$$

1 $s(x) = f(x) \cdot g(x)$

$$s'(x) = f(x)g'(x) + f'(x)g(x)$$
$$s'(1) = f(1)\,g'(1) + f'(1)\,g(1)$$

↑ slope of g when $x=1$ $\quad$ ↑ slope of f when $x=1$

$$s'(1) = (2)(-1) + (2)(1)$$
$$s'(1) = -2 + 2$$
$$s'(1) = \boxed{0}$$

16) a) $n(x) = \frac{f(x)}{g(x)}$ $\quad n'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

$$n'(2) = \frac{(3)(4) - (5)(4)}{3^2}$$

$$n'(2) = \boxed{\frac{-8}{9}}$$

b) $h(x) = f(g(x))$

$$h'(x) = f'(g(x)) \circ g'(x)$$

$$h'(1) = \underbrace{f'(g(1))} \circ g'(1)$$

$$= f'(2) \cdot g'(1)$$

$$= 4 \cdot 3$$

$$h'(1) = \boxed{12}$$

17) $f(x) = \sqrt[3]{(x^2-2x-1)^2} = (x^2-2x-1)^{2/3}$

Chain Rule

$$f'(x) = \frac{2}{3}(x^2-2x-1)^{-1/3} \cdot (2x-2)$$

$$f'(0) = \frac{2}{3}(0^2-2(0)-1)^{-1/3} \circ (2(0)-2)$$

$$= \frac{2}{3}(-1)(-2)$$

$$f'(0) = \boxed{\frac{4}{3}}$$

18) $h(x) = \begin{cases} x+3 & , x \le -2 \\ -x^2 & , x > -2 \end{cases}$

a) $h(-2) = -2+3 = 1$

h(-2) does exist

$h(-2) = 1$

b) $\lim_{x \to -2} h(x)$

$\lim_{h \to -2^-} h(x) = \lim_{h \to -2^+} h(x)$

The limit does not Exist

$1 \neq -4$

c) Does the limit $\lim_{x \to -2} h(x) = h(-2)$

DNE $\neq$ 1

Therefore it is Not continuous

19) $\lim_{x \to 2} \frac{x^2+3x-10}{x-2} = \lim_{x \to 2} \frac{(x+5)(x-2)}{x-2} = \lim_{x \to 2} x+5$

$= \boxed{7}$

20) $\lim_{x \to \infty} \frac{2x^3-3}{3x^3+25} = \boxed{\frac{2}{3}}$

Degrees are the same so we use the coefficients

21) a) Average rate of change = slope connecting points

$x=0$ $\qquad\qquad$ $x=3$

$f(0)=2(0)^2-5(0)$ $\qquad\qquad$ $f(3)=2(3)^2-5(3)$

$f(0)=0$ $\qquad\qquad$ $f(3)=3$

$(0,0)$ $\qquad\qquad$ $(3,3)$

$$\frac{3-0}{3-0}=\frac{3}{3}=\boxed{1} \leftarrow$$ Average rate of change

b) Instantaneous rate of change = Derivative

$f(x)=2x^2-5x$

$$\boxed{f'(x)=4x-5}$$

22) $f(x)=\frac{1}{4}x^4-\frac{1}{3}x^3-x^2$ $\qquad\qquad$ $f'(x)=x^3-x^2-2x$

$f(-2)=\frac{1}{4}(-2)^4-\frac{1}{3}(-2)^3-(-2)^2$ $\qquad\qquad$ $f'(-2)=(-2)^3-(-2)^2-2(-2)$

$f(-2)=\frac{8}{3}$ $\qquad\qquad$ $f'(2)=-8$

Point $\qquad\qquad$ Slope of normal $=\frac{1}{8}$

$\left(-2,\frac{8}{3}\right)$

$$\boxed{y-\frac{8}{3}=\frac{1}{8}(x+2)}$$